

Stimulated emission and Hawking radiation in black hole analogues

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Abstract. Stimulated emission by black holes is discussed in light of the analogue gravity program. We first consider initial quantum states containing a definite number of particles, and then we take into account the case where the initial state is a coherent state. The latter case is particularly significant in the case where Hawking radiation is studied in dielectric black holes, and the emission is stimulated by a laser probe. We are particularly interested in the case of the electromagnetic field, for which stimulated radiation is calculated too.

1. Introduction

Stimulated emission by black holes has been a longstanding topic in quantum field theory on a black hole background, which was taken into account since the very early studies in black hole evaporation [1, 2]. See also [3, 4]. Since the former analysis, it appeared that stimulated radiation is far from being of practical interest in the case of astrophysical black holes. Still, a relevant role for stimulated radiation is deserved in some attempts to solve the unitarity problem, because of the fact that stimulated radiation carries out information [5, 6]. We don't delve into the latter aspect. A different consideration for the same topic has to be deserved in the case of analogue gravity, because conditions where stimulated emission can play a very relevant role can be actually realized in labs. In particular, we are interested in dielectric black holes which are obtained as moving dielectric perturbations associated with strong laser pulses in nonlinear dielectric media via Kerr effect. Indeed, a further laser probe, of weak intensity (so not participating the Kerr effect) can be shot onto the dielectric perturbation, in such a way that an intense stimulated emission of pairs can occur. The conditions under which such stimulated emission is possible have been studied, both numerically and experimentally, by D.Faccio group in [7, 8, 9]. We limit ourselves to recall that the original idea of the Kerr effect as a tool for studying Hawking radiation in dielectric black holes can be traced back to Ref. [10], and a series of papers on the subject appeared in the literature [11, 12, 13, 14, 7, 15, 16, 17, 8, 18, 9, 19, 20].

Herein, we take into account some theoretical aspects of this stimulation phenomenon. We first follow strictly the presentation given in [3], which adopts the strategy of Bogoliubov transformations between IN and OUT states in a collapse situation. This strategy is well-known in black hole emission process since the seminal calculation by S.W.Hawking. As usual, a Fock space state with a definite number of particles is taken into account. We also provide a very simple deduction of the stimulated contribution to pair creation by means of thermofield dynamics formalism, which is particularly useful once more.

As a further matter of analysis, we take into consideration the case of a coherent state as initial state. This choice is, to some extent, on the opposite side with respect to the standard choice of a quantum state belonging to the Fock space, as it represents the best approximation to a classical state, to be compared with the eminently quantum nature of a Fock state. The coherent state stimulation appears of limited interest in the astrophysical black hole state, but is actually a very interesting topic in the case of dielectric black holes. Indeed, stimulated Hawking radiation can be obtained as described above, by means of a weak laser probe, whose nature of coherent light is well-known. This same strategy can be used also in the case of dielectric dispersive black holes, and we shall use it also for the standard ternary process which is at the root of Hawking radiation in the dielectric case.

We don't consider herein a full calculation for the full electromagnetic case, which

appears to be very involved, and limit our considerations to some general properties we expect to be implemented also in the full calculation, which is deferred to future works. The conclusions that we can infer for the standard ternary process $IN \rightarrow P + N$, where IN stays for the input particle state, and P and N stay for the particle and antiparticle final states, are the expected ones: the created pairs are such that one emitted photon is found in the P mode peak and the companion photon (antiparticle) is found in the N mode peak. The spontaneous contribution is unpolarized, as it should be due to thermality of the spontaneous radiation, whereas the stimulated one is suitably polarized, in such a way that a created photon is polarized in the same way as the stimulating particle or antiparticle state. As a consequence, given an IN state populated by particle states with a given polarization, one obtains a P mode peak and a N mode peak, all with the same polarization.

2. Stimulated emission and black holes

We start by summarizing some calculations appearing in [3]. Let $\{f_i\}$ be a set of positive norm solutions for the Klein-Gordon equation in the initial state labeled by IN (no black hole), and let us define $\{p_i\}$ as positive norm solutions available at infinity when a static black hole is present (label OUT). As known, to the latter set one has to join a further set of states $\{q_i\}$ which are not available to the distant observer (horizon states, label H) in order to get a basis for the solutions in presence of a static black hole. As both $\{f_i\}$ and $\{p_i\} \cup \{q_i\}$ represent a basis of solutions, one can e.g. write

$$p_i = \sum_j (\alpha_{ij} f_j + \beta_{ij} f_j^*), \quad (1)$$

$$q_i = \sum_j (\gamma_{ij} f_j + \eta_{ij} f_j^*), \quad (2)$$

together with

$$f_j = \sum_i (\alpha_{ij}^* p_i - \beta_{ij} p_i^* + \gamma_{ij}^* q_i - \eta_{ij} q_i^*). \quad (3)$$

As to the (charged) field, we get

$$\phi(x) = \sum_i (a_i^{IN} f_i + b_i^{IN \dagger} f_i^*) \quad (4)$$

$$= \sum_i (c_i^{OUT} p_i + d_i^{OUT \dagger} p_i^* + g_i^H q_i + h_i^{H \dagger} q_i^*), \quad (5)$$

where IN, OUT, H label the states in the initial condition, the outgoing modes in presence of black hole and the horizon states respectively. As to the relations between creation-annihilation operators in the different bases, we are interested in

$$c_i^{OUT} = \sum_j (\alpha_{ij}^* a_j^{IN} - \beta_{ij}^* b_j^{IN \dagger}), \quad (6)$$

$$d_i^{OUT \dagger} = \sum_j (\alpha_{ij} b_j^{IN \dagger} - \beta_{ij} a_j^{IN}), \quad (7)$$

and conjugate ones. The number of particles operator for the mode k is

$$\begin{aligned} N_k^{OUT} &= c_k^{OUT} \dagger c_k^{OUT} \\ &= \sum_{jl} (\alpha_{kj} \alpha_{kl}^* a_j^{IN} \dagger a_l^{IN} - \alpha_{kj} \beta_{kl}^* a_j^{IN} \dagger b_l^{IN} \dagger \\ &\quad - \beta_{kj} \alpha_{kl}^* b_j^{IN} a_l^{IN} + \beta_{kj} \beta_{kl}^* b_j^{IN} b_l^{IN} \dagger). \end{aligned} \quad (8)$$

Spontaneous pair creation occurs when

$$\langle 0_{IN} | N_k^{OUT} | 0_{IN} \rangle = \sum_j |\beta_{kj}|^2 \quad (9)$$

is different from zero. Stimulated emission occurs if there are particles (and/or antiparticles) in the initial state: let

$$|\psi_{IN}\rangle \in \mathcal{F}, \quad (10)$$

i.e. let the initial state belong to the Fock space \mathcal{F} and be an eigenstate of the number operator N^{IN} . Then, it is straightforward to show that

$$\begin{aligned} \langle \psi_{IN} | N_k^{OUT} | \psi_{IN} \rangle &= \sum_j |\beta_{kj}|^2 + \sum_j |\alpha_{kj}|^2 \langle n_j^{IN} \rangle \\ &\quad + \sum_j |\beta_{kj}|^2 \langle \bar{n}_j^{IN} \rangle, \end{aligned} \quad (11)$$

where n_j^{IN} stays for the number of particles in the j -state IN and \bar{n}_j^{IN} stays for the number of antiparticles in the j -state IN. As it is evident, the first contribution is associated again with the spontaneous pair creation, whereas the two further contributions are associated with the particle and antiparticle content of the initial state, and represent stimulated pair creation contributions.

For the antiparticle number operator, we have

$$\begin{aligned} \bar{N}_k^{OUT} &= d_k^{OUT} \dagger d_k^{OUT} \\ &= \sum_{jl} (\alpha_{kj} \alpha_{kl}^* b_j^{IN} \dagger b_l^{IN} - \alpha_{kj} \beta_{kl}^* b_j^{IN} \dagger a_l^{IN} \dagger \\ &\quad - \beta_{kj} \alpha_{kl}^* a_j^{IN} b_l^{IN} + \beta_{kj} \beta_{kl}^* a_j^{IN} a_l^{IN} \dagger). \end{aligned} \quad (12)$$

so that

$$\langle 0_{IN} | \bar{N}_k^{OUT} | 0_{IN} \rangle = \sum_j |\beta_{kj}|^2, \quad (13)$$

and

$$\begin{aligned} \langle \psi_{IN} | \bar{N}_k^{OUT} | \psi_{IN} \rangle &= \sum_j |\beta_{kj}|^2 + \sum_j |\alpha_{kj}|^2 \langle \bar{n}_j^{IN} \rangle \\ &\quad + \sum_j |\beta_{kj}|^2 \langle n_j^{IN} \rangle. \end{aligned} \quad (14)$$

In the case of diagonal Bogoliubov transformations, we obtain

$$\langle \psi_{IN} | N_k^{OUT} | \psi_{IN} \rangle = |\beta_k|^2 + |\alpha_k|^2 \langle n_k^{IN} \rangle + |\beta_k|^2 \langle \bar{n}_k^{IN} \rangle, \quad (15)$$

and

$$\langle \psi_{IN} | \bar{N}_k^{OUT} | \psi_{IN} \rangle = |\beta_k|^2 + |\alpha_k|^2 \langle \bar{n}_k^{IN} \rangle + |\beta_k|^2 \langle n_k^{IN} \rangle. \quad (16)$$

It is worth noting that the above formulas hold true also in a more generic case where one passes from an IN state to a OUT one. Indeed, the horizon states do not affect the results for what concerns the number of particles measured at infinity.

According to the calculations in [2], which amount substantially to the ones obtained in [3] by means of a careful wave packet analysis in a collapse situation, one finds in the black hole case

$$\langle \psi_{IN} | N_{\omega}^{OUT} | \psi_{IN} \rangle = \frac{1}{e^{\beta\omega} - 1} + \frac{e^{\beta\omega}}{e^{\beta\omega} - 1} \langle n_{\omega}^{IN} \rangle + \frac{1}{e^{\beta\omega} - 1} \langle \bar{n}_{\omega}^{IN} \rangle, \quad (17)$$

and

$$\langle \psi_{IN} | \bar{N}_{\omega}^{OUT} | \psi_{IN} \rangle = \frac{1}{e^{\beta\omega} - 1} + \frac{e^{\beta\omega}}{e^{\beta\omega} - 1} \langle \bar{n}_{\omega}^{IN} \rangle + \frac{1}{e^{\beta\omega} - 1} \langle n_{\omega}^{IN} \rangle, \quad (18)$$

where β is the inverse black hole temperature. It is interesting to note that, when the stimulated effect dominates over the spontaneous one, and in absence of initial antiparticles, one obtains

$$\langle \psi_{IN} | N_{\omega}^{OUT} | \psi_{IN} \rangle \sim \frac{e^{\beta\omega}}{e^{\beta\omega} - 1} \langle n_{\omega}^{IN} \rangle. \quad (19)$$

It is worth noting that stimulated emission can be calculated also by assuming as thermal state the Hartle-Hawking one, and by exploiting the relations between Hartle-Hawking state and Thermofield Dynamics as in [25]. The calculation is straightforward, and proceeds as follows. We recall that in the thermofield dynamics formalism one defines a thermal state $|0(\beta)\rangle$ characterized by an inverse temperature β . This state requires a doubling of degrees of freedom with respect to the standard quantum field theory at zero temperature, and the (unobservable) copy of the fictitious Hilbert space is characterized by operators with a tilde symbol. The thermal state $|0(\beta)\rangle$ is annihilated by suitable operators $a_l(\beta), \tilde{a}_l(\beta), b_l(\beta), \tilde{b}_l(\beta)$ (and conjugated ones) which are labeled by a complete set of quantum numbers l and is related to “standard” annihilation-creation operators $a_l, \tilde{a}_l, b_l, \tilde{b}_l$ (and conjugated ones) via a formally unitary transformation:

$$a_l = \cosh(\phi_{\omega_l}) a_l(\beta) + \sinh(\phi_{\omega_l}) \tilde{a}_l^{\dagger}(\beta), \quad (20)$$

$$\tilde{a}_l^{\dagger} = \sinh(\phi_{\omega_l}) a_l(\beta) + \cosh(\phi_{\omega_l}) \tilde{a}_l^{\dagger}(\beta), \quad (21)$$

and analogous for b -operators, with

$$\tanh(\phi_{\omega_l}) = e^{-\beta\omega_l/2}. \quad (22)$$

It holds

$$|0(\beta)\rangle = e^{-iG} |0, \tilde{0}\rangle, \quad (23)$$

where

$$G = \sum_l \log(\tanh(\beta\omega_l)) \left[(\tilde{a}_l a_l - \tilde{a}_l^{\dagger} a_l^{\dagger}) + (\tilde{b}_l b_l - \tilde{b}_l^{\dagger} b_l^{\dagger}) \right]. \quad (24)$$

In the following, we indicate by HH the Hartle-Hawking state and by B the Boulware state in the case of Schwarzschild black hole, and R and L are the labels for the right and left region of the Kruskal diagram, as usual. L is the label for unobservable states,

of course. For simplicity, we consider a neutral scalar field (and then only a -modes appear). Moreover, we indicate with $|B\rangle$ the state $|B, \tilde{B}\rangle$. We get

$$|HH\rangle = \exp(-iG)|B\rangle = \exp\left[\sum_{\omega}\left(\frac{1}{2}\log(1 - \exp(-\frac{\pi\omega}{2\kappa}))\right)\right] \exp\left[\exp(-\frac{\pi\omega}{\kappa})\sum_j\left(a_{\omega j}^{R\dagger}a_{\omega j}^{L\dagger}\right)\right]|B\rangle. \quad (25)$$

We have also

$$|HH\rangle = \frac{1}{Z(\kappa)} \prod_{\omega j} \left(\sum_{n_{\omega j}=0}^{\infty} \exp\left(-n_{\omega j} \frac{\pi\omega}{\kappa}\right) \right) |n_{\omega j} >^L |n_{\omega j} >^R. \quad (26)$$

States $|n_{\omega j} >^L, |n_{\omega j} >^R$ are Boulware states with $n_{\omega j}$ particles. Killing observers in the R-region cannot measure L -states, so a trace over the latter ones is required. As a consequence, for the expectation value of an operator (observable) A^R , constructed by means of Boulware vacuum creation and annihilation operators, one obtains

$$\langle HH|A^R|HH\rangle = \frac{1}{Z(\kappa)} \sum_n \exp(-\beta_H^* E_n) \langle n|A^R|n\rangle, \quad (27)$$

where $E_n = \frac{1}{K} \sum_{\omega j} n_{\omega j} \omega$, $\beta_H^* = \beta_H K$ (K stays for the norm of the timelike Killing vector), $Z = \sum_n \exp(-\beta_H^* E_n)$. The relation with thermofield dynamics is then implemented, with the identification of the fictitious states with the states in the inaccessible L -region. In particular, we get

$$\langle HH|N_k^R|HH\rangle = \langle HH|a_k^{R\dagger}a_k^R|HH\rangle = \frac{1}{e^{\beta\omega_k} - 1}, \quad (28)$$

as expected.

Thermofield dynamics formalism allows us to set out the following identifications:

$$\alpha_k = \cosh(\phi_{\omega_k}), \quad (29)$$

$$\beta_k = \sinh(\phi_{\omega_k}), \quad (30)$$

i.e. the Bogoliubov transformation is diagonal and, moreover, real valued. According to the definition in [26], we can introduce thermal number states as follows (for simplicity, we refer to a single mode of the field):

$$|n(\beta)\rangle = \frac{1}{\sqrt{n!}} (a^\dagger(\beta))^n |0(\beta)\rangle, \quad n = 0, 1, 2, \dots \quad (31)$$

Stimulated emission can be obtained by calculating the mean value of N_k, \bar{N}_k on thermal number states: we define

$$\langle A_k \rangle_{n, \bar{n}, \beta} := \langle n_k(\beta), \bar{n}_k(\beta) | A_k | n_k(\beta), \bar{n}_k(\beta) \rangle, \quad (32)$$

then we obtain

$$\langle N_k \rangle_{n, \bar{n}, \beta} = \sinh^2(\phi_{\omega_k}) + \cosh^2(\phi_{\omega_k}) n_k(\beta) + \sinh^2(\phi_{\omega_k}) \bar{n}_k(\beta), \quad (33)$$

$$\langle \bar{N}_k \rangle_{n, \bar{n}, \beta} = \sinh^2(\phi_{\omega_k}) + \sinh^2(\phi_{\omega_k}) n_k(\beta) + \cosh^2(\phi_{\omega_k}) \bar{n}_k(\beta), \quad (34)$$

which correspond to the above mentioned results for stimulated emission by black holes. Note that

$$\sinh^2(\phi_{\omega_k}) = \frac{1}{e^{\beta\omega_k} - 1}, \quad (35)$$

$$\cosh^2(\phi_{\omega_k}) = \frac{e^{\beta\omega_k}}{e^{\beta\omega_k} - 1}. \quad (36)$$

Stimulated emission appears as the mean value of the number operator over an ‘initial’ state which is a thermofield state with non-zero thermal particles/antiparticles. In a black hole context, we have non-zero Hartle-Hawking particles stimulating the emission process. A more direct physical intuition can be provided by the Unruh effect situation: non-zero particles as excitations of the Minkowski vacuum stimulate the thermal emission for the Rindler observer.

3. Stimulated emission and initial coherent states

In the previous section, we discussed what happens if the initial state is characterized by a definite (and finite) number of particles and/or antiparticles. These states are standard in quantum field theory calculations, but don’t cover exhaustively the possibilities one could have in mind. For example, one could take into account coherent states, which correspond to field states as near as possible to classical states of the fields. This consideration of coherent states, which appears to be of purely speculative interest in the case of black holes, is instead of great interest in case of black hole analogues. Indeed, in the case of dielectric black holes which are generated by means of the Kerr effect, a dielectric perturbation traveling in the dielectric medium plays the role of the black hole, and is generated by a strong laser pulse. When a further weak laser probe is shot against the probe pulse, one can obtain stimulated emission. A formal way to describe the laser probe is by means of coherent states, which makes very strong and worthwhile of direct physical application the interest in this topic.

We recall that a coherent state is defined as an eigenstate of the annihilation operator. For example, if \hat{c} is an annihilation operator, such that

$$\hat{c}|0\rangle = 0, \quad (37)$$

and $[\hat{c}, \hat{c}^\dagger] = 1$, then a coherent state $|z\rangle$ is such that

$$\hat{c}|z\rangle = z|z\rangle, \quad (38)$$

and also

$$\begin{aligned} |z\rangle &= e^{-|z|^2/2} \sum_{n=0}^{\infty} \frac{z^n}{n!} |n\rangle \\ &= e^{z\hat{c}^\dagger - z^*\hat{c}} |0\rangle, \end{aligned} \quad (39)$$

where $|n\rangle = \hat{c}^{\dagger n}|0\rangle$ has norm $\sqrt{n!}$. We recall also that a Poissonian distribution of particles corresponds to the coherent state, and also that coherent states can be normalized (we assume that ours ones in the following are normalized) but not

orthogonal each other, and form an overcomplete set of states. Moreover, in a field theory framework, we should write [24]

$$|\eta\rangle = e^{\int \frac{d^3\mathbf{k}}{(2\pi)^3 2\omega_k} (\eta(\mathbf{k}) \hat{c}^\dagger(\mathbf{k}) - \eta^*(\mathbf{k}) \hat{c}(\mathbf{k}))} |0\rangle, \quad (40)$$

with

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3 2\omega_k} |\eta(\mathbf{k})|^2 < \infty. \quad (41)$$

We recall that

$$\hat{c}(\mathbf{k})|\eta\rangle = \eta(\mathbf{k})|\eta\rangle. \quad (42)$$

However, in the following, we shall keep the discrete notation, in order to simplify the presentation. Like in the previous section, we will work with a charged scalar field, obviously all results being immediately adaptable also to the case of neutral scalar field, in a straightforward way. In the charged field two distinct annihilation operator types appear: the a -type for particles and the b -type for antiparticles in the initial state field modes (and c -type and d -type for the outgoing field modes). In line of principles, we could consider an initial coherent state constructed as the tensor product of $|\eta^{IN}\rangle$ and $|\bar{\eta}^{IN}\rangle$, where

$$a_i^{IN}|\eta^{IN}\rangle = \eta_i|\eta^{IN}\rangle, \quad (43)$$

$$b_i^{IN}|\bar{\eta}^{IN}\rangle = \bar{\eta}_i|\bar{\eta}^{IN}\rangle, \quad (44)$$

(we have omitted the suffix IN where no confusion is possible), so that the initial state is simply

$$|\zeta^{IN}\rangle = |\eta^{IN}\rangle \otimes |\bar{\eta}^{IN}\rangle. \quad (45)$$

We get

$$\begin{aligned} \langle \zeta^{IN} | N_k^{OUT} | \zeta^{IN} \rangle &= \sum_j |\beta_{kj}|^2 + \sum_{jl} (\alpha_{kj} \alpha_{kl}^* \eta_j^* \eta_l - \alpha_{kj} \beta_{kl}^* \eta_j^* \bar{\eta}_l^* \\ &\quad - \beta_{kj} \alpha_{kl}^* \bar{\eta}_j \eta_l + \beta_{kj} \beta_{kl}^* \bar{\eta}_j \bar{\eta}_l^*). \end{aligned} \quad (46)$$

Spontaneous pair creation contribution is the first term on the right side, and is the usual one occurring in absence of incoming particles/antiparticles. The remaining terms are stimulated contributions. Analogously,

$$\begin{aligned} \langle \zeta^{IN} | \bar{N}_k^{OUT} | \zeta^{IN} \rangle &= \sum_j |\beta_{kj}|^2 + \sum_{jl} (\alpha_{kj} \alpha_{kl}^* \bar{\eta}_j^* \bar{\eta}_l - \alpha_{kj} \beta_{kl}^* \bar{\eta}_j^* \eta_l^* \\ &\quad - \beta_{kj} \alpha_{kl}^* \eta_j \bar{\eta}_l + \beta_{kj} \beta_{kl}^* \eta_j \eta_l^*). \end{aligned} \quad (47)$$

We note that, in absence of initial antiparticle states, we obtain $|\zeta^{IN}\rangle = |\eta^{IN}\rangle \otimes |0\rangle$, and then

$$\langle \zeta^{IN} | N_k^{OUT} | \zeta^{IN} \rangle = \sum_j |\beta_{kj}|^2 + \sum_{jl} \alpha_{kj} \alpha_{kl}^* \eta_j^* \eta_l \quad (48)$$

and

$$\langle \zeta^{IN} | \bar{N}_k^{OUT} | \zeta^{IN} \rangle = \sum_j |\beta_{kj}|^2 + \sum_{jl} \beta_{kj} \beta_{kl}^* \eta_j \eta_l^*. \quad (49)$$

With respect to the case where a Fock initial state is considered, we face with a situation where nondiagonal contributions appear.

In the diagonal case, we obtain

$$\langle \zeta^{IN} | N_k^{OUT} | \zeta^{IN} \rangle = |\beta_k|^2 + |\alpha_k|^2 |\eta_k|^2 \quad (50)$$

$$\langle \zeta^{IN} | \bar{N}_k^{OUT} | \zeta^{IN} \rangle = |\beta_k|^2 + |\beta_k|^2 |\eta_k|^2. \quad (51)$$

By choosing the diagonal Bogoliubov transformation corresponding to the passage from Hartle-Hawking modes to Boulware ones thermality emerges again. It is also interesting to point out that (51) coincides with (15) under the same hypothesis of zero initial antiparticles, due to the fact that $|\eta_k|^2 = \langle N_k^{IN} \rangle$. In order to distinguish between the two different situations it is useful to calculate the variance

$$(\delta^2 N_k^{OUT})_\varphi = \langle \varphi^{IN} | (N_k^{OUT})^2 | \varphi^{IN} \rangle - \langle \varphi^{IN} | N_k^{OUT} | \varphi^{IN} \rangle^2, \quad (52)$$

where φ^{IN} is the initial state of interest.

A bit long calculation shows that variance is different in the two cases, and provides us a simple tool for discriminating between the aforementioned physical situations.

We first take into account the Fock space case. We note that the presence of a Bogoliubov transformation makes nontrivial the result, in the sense that, even if trivially we have $\langle \psi_{IN} | (N_k^{IN})^2 | \psi_{IN} \rangle - \langle \psi_{IN} | N_k^{IN} | \psi_{IN} \rangle^2 = 0$, one obtains

$$\begin{aligned} (\delta^2 N_k^{OUT})_\psi &= \langle \psi^{IN} | (N_k^{OUT})^2 | \psi^{IN} \rangle - \langle \psi^{IN} | N_k^{OUT} | \psi^{IN} \rangle^2 \\ &= \sum_{ij} [|\alpha_{ki}|^2 |\alpha_{kj}|^2 (\langle n_i^{IN} \rangle + 1) \langle n_j^{IN} \rangle \\ &\quad + |\beta_{ki}|^2 |\beta_{kj}|^2 \langle \bar{n}_i^{IN} \rangle (\langle \bar{n}_j^{IN} \rangle + 1) \\ &\quad + |\beta_{ki}|^2 |\alpha_{kj}|^2 \langle n_i^{IN} \rangle \langle \bar{n}_j^{IN} \rangle \\ &\quad + |\alpha_{ki}|^2 |\beta_{kj}|^2 (\langle n_i^{IN} \rangle + 1) (\langle \bar{n}_j^{IN} \rangle + 1)] . \end{aligned} \quad (53)$$

In absence of initial antiparticle states we find

$$\begin{aligned} (\delta^2 N_k^{OUT})_\psi &= \sum_{ij} [|\alpha_{ki}|^2 |\alpha_{kj}|^2 (\langle n_i^{IN} \rangle + 1) \langle n_j^{IN} \rangle \\ &\quad + |\alpha_{ki}|^2 |\beta_{kj}|^2 (\langle n_i^{IN} \rangle + 1)] . \end{aligned} \quad (54)$$

In the diagonal case one obtains

$$(\delta^2 N_k^{OUT})_\psi = |\alpha_k|^4 (\langle n_k^{IN} \rangle + 1) \langle n_k^{IN} \rangle + |\alpha_k|^2 |\beta_k|^2 (\langle n_k^{IN} \rangle + 1). \quad (55)$$

In the coherent state representation we have a non vanishing variance on the number of outgoing particles too. A direct computation gives

$$\begin{aligned} (\delta^2 N_k^{OUT})_\zeta &= \langle \zeta^{IN} | (N_k^{OUT})^2 | \zeta^{IN} \rangle - \langle \zeta^{IN} | N_k^{OUT} | \zeta^{IN} \rangle^2 \\ &= \sum_j |\alpha_{kj}|^2 \sum_i |\beta_{ki}|^2 \\ &\quad + \left[\sum_j (|\alpha_{kj}|^2 + |\beta_{kj}|^2) \right] \left| \sum_i (\alpha_{ki} \eta_i^* - \beta_{ki} \bar{\eta}_i) \right|^2. \end{aligned} \quad (56)$$

In absence of initial antiparticle states it becomes

$$(\delta^2 N_k^{OUT})_\zeta = \sum_j |\alpha_{kj}|^2 \sum_i |\beta_{ki}|^2 + \left[\sum_j (|\alpha_{kj}|^2 + |\beta_{kj}|^2) \right] \left| \sum_i \alpha_{ki} \eta_i^* \right|^2. \quad (57)$$

In the diagonal case, we get

$$(\delta^2 N_k^{OUT})_\zeta = |\alpha_k|^2 |\beta_k|^2 + (|\alpha_k|^2 + |\beta_k|^2) |\alpha_k|^2 |\eta_k|^2. \quad (58)$$

Summarizing, in absence of initial antiparticles and in the diagonal case, we find in the Fock state case

$$\begin{aligned} (\delta^2 N_k^{OUT})_\psi &= |\alpha_k|^4 \langle n_k^{IN} \rangle^2 + |\alpha_k|^4 \langle n_k^{IN} \rangle \\ &\quad + |\alpha_k|^2 |\beta_k|^2 \langle n_k^{IN} \rangle + |\alpha_k|^2 |\beta_k|^2, \end{aligned} \quad (59)$$

to be compared with the coherent state case

$$(\delta^2 N_k^{OUT})_\zeta = |\alpha_k|^4 \langle n_k^{IN} \rangle + |\beta_k|^2 |\alpha_k|^2 \langle n_k^{IN} \rangle + |\alpha_k|^2 |\beta_k|^2. \quad (60)$$

Then we find

$$(\delta^2 N_k^{OUT})_\psi - (\delta^2 N_k^{OUT})_\zeta = |\alpha_k|^4 \langle n_k^{IN} \rangle^2. \quad (61)$$

As a consequence, variance is a good tool for discriminating between the Fock state case and the coherent state one, when measurements of the expectation value of N_k^{OUT} is not decisive, as shown above.

4. Stimulated emission and the electromagnetic field

The presence of the electromagnetic field is made more involved because of the gauge invariance, requiring a suitable gauge fixing, and the appearance, in the case e.g. of a covariant gauge, of spurious degrees of freedom (scalar component and longitudinal component of the field) to be suitably taken into account. We do not delve into a detailed discussion (for the quantization of the electromagnetic field on a manifold and in black hole backgrounds see e.g. [21, 22, 23]), we limit ourselves to some basic considerations which still shed light on stimulated emission in the case of the electromagnetic field.

Our key-ansatz consists in assuming that, e.g. in a generalized Feynman gauge [22] it is possible to separate two mutually orthogonal physical polarizations $\lambda = 1, 2$ in such a way that they remain orthogonal to scalar and longitudinal polarizations. This is ensured in the case of dielectric black holes, because asymptotic states are well-known polariton states of standard flat spacetime [19, 27, 28]. The polarization index appears as a further label to be considered together with quantum number specifying solutions of the homogeneous Maxwell equations in presence of a black hole. In this sense, there are no substantial changes with respect to the (neutral) scalar field case, for what concerns the calculation of the Bogoliubov coefficients. Indeed, one has e.g.

$$\beta_{\lambda k, \lambda' k'} = -(p_\mu^{\lambda' k'}, f_\mu^{\lambda k*}), \quad (62)$$

in a straightforward generalization of the scalar field case. k stays for the set of labels specifying solutions of the Maxwell equations, and it is interesting to stress that we are only interested to the cases where $\lambda, \lambda' = 1, 2$, i.e. only physical polarizations have to be taken into account in order to calculate physical observables. Indeed, the remaining

polarizations appear only in the set of unphysical variables, and do not participate any physically relevant number operator. Then we obtain

$$\begin{aligned} \langle \psi_{IN} | N_{\lambda k}^{OUT} | \psi_{IN} \rangle &= \sum_{\lambda' j} |\beta_{\lambda k, \lambda' j}|^2 + \sum_{\lambda' j} |\alpha_{\lambda k, \lambda' j}|^2 \langle n_{\lambda' j}^{IN} \rangle \\ &+ \sum_{\lambda' j} |\beta_{\lambda k, \lambda' j}|^2 \langle \bar{n}_{\lambda' j}^{IN} \rangle, \end{aligned} \quad (63)$$

where $\lambda, \lambda' = 1, 2$. One can obtain straightforwardly an analogous equation for $\bar{N}_{\lambda k}^{OUT}$. In particular, in the thermal particle creation case one finds for the physical polarizations

$$\begin{aligned} \langle \psi_{IN} | N_{\lambda \omega}^{OUT} | \psi_{IN} \rangle &= \frac{1}{e^{\beta \omega} - 1} + \frac{e^{\beta \omega}}{e^{\beta \omega} - 1} \langle n_{\lambda \omega}^{IN} \rangle \\ &+ \frac{1}{e^{\beta \omega} - 1} \langle \bar{n}_{\lambda \omega}^{IN} \rangle, \end{aligned} \quad (64)$$

$$\begin{aligned} \langle \psi_{IN} | \bar{N}_{\lambda \omega}^{OUT} | \psi_{IN} \rangle &= \frac{1}{e^{\beta \omega} - 1} + \frac{e^{\beta \omega}}{e^{\beta \omega} - 1} \langle \bar{n}_{\lambda \omega}^{IN} \rangle \\ &+ \frac{1}{e^{\beta \omega} - 1} \langle n_{\lambda \omega}^{IN} \rangle. \end{aligned} \quad (65)$$

As expected, the polarization does not affect spontaneous emission, which is clearly a unpolarized contribution. The two further terms, instead, depend on λ as far as the initial particle and/or antiparticle states are polarized. If we assume that zero antiparticles are present in the initial state, then,

$$\langle \psi_{IN} | N_{\lambda \omega}^{OUT} | \psi_{IN} \rangle = \frac{1}{e^{\beta \omega} - 1} + \langle n_{\lambda \omega}^{IN} \rangle + \frac{1}{e^{\beta \omega} - 1} \langle n_{\lambda \omega}^{IN} \rangle, \quad (66)$$

$$\langle \psi_{IN} | \bar{N}_{\lambda \omega}^{OUT} | \psi_{IN} \rangle = \frac{1}{e^{\beta \omega} - 1} + \frac{1}{e^{\beta \omega} - 1} \langle n_{\lambda \omega}^{IN} \rangle. \quad (67)$$

The first contribution on the right side of both the above equations is the spontaneous (unpolarized) contribution. The second one in (66) is the number of photons in the IN state, and the third one in the same equation is the stimulated (polarized) contribution. In (66) we find, beyond the spontaneous contribution, a further polarized stimulated term. In the limit for $n_{\lambda \omega}^{IN} \gg 1$, one finds

$$\langle \psi_{IN} | N_{\lambda \omega}^{OUT} | \psi_{IN} \rangle \sim \langle n_{\lambda \omega}^{IN} \rangle + \frac{1}{e^{\beta \omega} - 1} \langle n_{\lambda \omega}^{IN} \rangle, \quad (68)$$

$$\langle \psi_{IN} | \bar{N}_{\lambda \omega}^{OUT} | \psi_{IN} \rangle \sim \frac{1}{e^{\beta \omega} - 1} \langle n_{\lambda \omega}^{IN} \rangle. \quad (69)$$

and

$$\langle \psi_{IN} | \bar{N}_{\lambda \omega}^{OUT} | \psi_{IN} \rangle \sim \langle \psi_{IN} | N_{\lambda \omega}^{OUT} | \psi_{IN} \rangle \quad (70)$$

only if $e^{\beta \omega} - 1 \ll 1$.

It is straightforward to generalize to the electromagnetic case our analysis involving initial coherent states. We limit ourselves to the general formula for physical polarizations:

$$\langle \zeta^{IN} | N_{\lambda k}^{OUT} | \zeta^{IN} \rangle = \sum_{\lambda' j} |\beta_{\lambda k, \lambda' j}|^2 + \sum_{\lambda' j, \lambda'' l} (\alpha_{\lambda k, \lambda' j} \alpha_{\lambda k, \lambda'' l}^* \eta_{\lambda' j}^* \eta_{\lambda'' l})$$

Let us interpret the above results in terms of the observed P -peak (particle peak) and N -peak (antiparticle peak). Our considerations are limited to the case where thermality occurs and no relevant contribution to the scattering process of the fourth mode involved in the process, called B -mode, appears. (66) represents the mean number of photons in the P -peak, whereas (67) stays for the number of antiparticles in the N -peak. As it is evident, in the P -peak we find the initial state photons (i.e. photons in the state IN), and, as an effect of the thermal particle creation, of the stimulated and spontaneous created particles. In the N -peak, we find stimulated and spontaneous created antiparticles. We have that the pair-created photons participate both the peaks, the particle modes joining the P -peak together with the IN -photons, and the antiparticle modes forming the N -peak, which represents a clear signal of amplification (pair creation). As to the polarization, we note that both the peaks, as far as the stimulated contribution becomes the leading one, are polarized. In particular, polarized IN -photons generate particles and antiparticles with the same polarization as for the IN -photons.‡

5. Conclusions

We have reconsidered stimulated emission in a black hole context, in light of the fact that analogue gravity allows a relevant role for stimulated emission, in spite of its substantial practical irrelevance for astrophysical black holes. We have recalled existing results for black holes, and we have pointed out as, in the case of Hartle-Hawking state, stimulated emission can be calculated in a thermofield dynamics framework. Then, we have considered coherent states as possible initial states, in view of the possibility in a dielectric black hole case to stimulate Hawking-like emission by means of a laser probe. Then we have also extended the results to the case of electromagnetic field, which, under reasonable hypotheses, are (non-trivial) extensions of the standard scalar field results. Finally, we have applied our analysis to the dielectric black hole case. The created pairs are such that one emitted photon is found in the P mode peak and the companion photon (antiparticle) is found in the N mode peak. The spontaneous contribution is unpolarized, as it should be due to thermality of the spontaneous radiation, whereas the stimulated one is suitably polarized, in such a way that a created photon is polarized in the same way as the stimulating particle state, and is polarized in the opposite way as the stimulating antiparticle state. As a consequence, given an IN state populated by particle states, one obtains a P mode peak and an N mode peak, all having the same polarization.

‡ Obviously particles and antiparticles initially have opposite phases, but in terms of experimental observation this fact is irrelevant since in general the phases will change independently along the story of the single particle.

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